Hidden Teacher's Incentives in Teaching Mathematics (a Case Study)

Abstract:
The paper studies types of teacher's hidden incentives (in Brousseau's Theory of Didactical Situations called Topaze effect) in teaching mathematics. The method used is the method of multiple case study. Transcripts of communication between two teachers (from two schools) and their 14-15 year old pupils in ten consecutive lessons of mathematics are analysed. Several types of explicit (overt) and implicit (covert) teacher's prompting are detected. The study characterizes and illustrates these types of a teacher's behaviour (Topaze effect) using examples of classroom dialogue and makes conjectures about their reasons. The influence of Topaze effect on quality of pupils' understanding of mathematics and cultivation of (mathematical) thinking is discussed.

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Introduction

Theoretical background: Theory of Didactical Situations in Mathematics (Guy Brousseau and his collaborators)

Didactical contract for explaining environment effects on learning mathematics

The set of the teacher's behaviours (specific to the taught knowledge) expected by the student and the set of the student's behaviour expected by the teacher

Not a real contract, never contracted either explicitly or implicitly between the contractors.

Criteria of satisfaction can never be really précised. (Brousseau

Unwanted effects and developments that can be observed: **Topaze effect**, Jourdain effect, metacognitive shift, improper use of analogy

Topaze effect (TE)

When the teacher wants his/her pupils to be active (find an answer on their own) and they cannot, the teacher disguises the expected answer or performance by various behaviours or attitudes, without providing the answer explicitly.

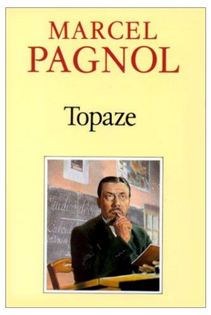
In order to help the pupil to give the expected answer, the teacher 'suggests' the answer, hiding it behind progressively more transparent didactical coding.

The knowledge necessary to produce the answer changes
- lowering the level of intellectual demand of the task.

It is a reaction, an action or an answer that is expected from students.

Understanding is not checked.

The teacher replaces explanation by a hint.



Materials and methods

Design of the study: qualitative methodology, multiple case study

Data analysis

Firth analysis of data (occurrence of TE) → Characteristics of TE, confrontation with data → List of types of TE

Data

Framework: Learner's Perspective Study (LPS) (Clarke et al., 2006)

Use of three video-cameras in the classroom supplemented by immediate post-lesson video-stimulated interviews Teaching of sequences of 10 lessons (enables to take into account the teacher's purposeful selection of instructional strategies)

Data gathered in the 8th grade (age 14-15) of a junior secondary school

Two schools, experienced teachers

Results

Types of Topaze Effect

Explicit (overt) teacher's prompting can be of the following nature:
- explanation of steps which pupils are expected to follow, although the procedure is obvious/well known,
- questions anticipating/prompting the subsequent solving procedure, pupils are not expected to answer,
- warning on a possible mistake,
- recollection of previous experience or knowledge e.g. by pointing out an analogy, either with a problem type or with a previously solved problem, although it is obvious/well known.

Implicit (covert) teacher's prompting can be of the following nature:
- rephrasing of their own or pupils' statements,
- use of signal words,
- saying the beginning of words,
- asking questions that lead to a simplification of the solving process,
- doubting correctness in situations where the pupil's answer is not correct or suitable.

Illustration of explanation of steps which pupils are expected to follow (TE in blue)

Teacher: Yes. What will the first number look like, Adam?
Adam: x. [*T writes on the blackboard: 1st number …… x.*]
T: Well done. And what will the second number look like, David?
David: x + 1 [*T writes on the blackboard: 2nd number …… x + 1.*]
T: Well done. So. And now, Lenka, make an equation from it, Lenka.
Lenka: x + x + 1
T: This is the sum of two consecutive natural numbers. [*T writes on the blackboard: x + x + 1 …*]
Lenka: 3x
T: And of their triples, yes. [*T writes on the blackboard: … + 3x + 3(x + 1) …*]
T: You should say plus.
Lenka: … 3x + 3(x + 1)
T: Correct.

The task: The sum of two consecutive natural numbers and their triples is 92. What are the numbers?

Conclusion

The teachers use different types of hints leading to the Topaze effect quite naturally and equally the pupils naturally expect it (built into the didactic contract). Teachers use them usually not quite consciously, as they are rooted in teachers' deeper pedagogical belief about the help that pupils need during lessons (not only) of mathematics.
Solution:
- Pay attention to quality of questions, so called 'good' questions
- Inquiry based (mathematics) education
- Individual problem solving
- Teachers´ reaction on pupils´ mistakes (recognition and elimination)
- Support of pupils' competence to solve the problems individually and to judge the correctness of a solution.

References

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